St Stephen's School – Carramar Campus
Year 11 Mathematics Specialist 3A Test 1
Total Marks: 50 Time Allowed: 60 mins Resource Rich

Name: Solutions **Mark:**

Teacher:________________________________

Parent Signature:

INSTRUCTIONS

Permitted equipment:

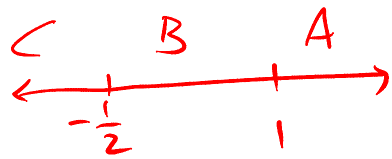
- Two calculators complying with Curriculum Council requirements
- Two A4 pages (both sides) of notes
- Stationery and drawing equipment

Unless otherwise specified, all answers should be rounded to two decimal places.

Question 1. **[9 marks]**

Solve algebraically, showing full working:

$$|x-1|+1=|2x+1|-6$$



$|x-1| = |2x+1| - 7$

A: $x \geq 1$ ✓

$x-1 = 2x+1-7$
$-x = -5$
$\underline{x = 5}$ ✓✓

B: $-\frac{1}{2} \leq x < 1$ ✓

$-(x-1) = 2x+1-7$
$-x+1 = 2x-6$
$-3x = -7$ ✓✓
$x = 7/3$ ✗ Not in part B

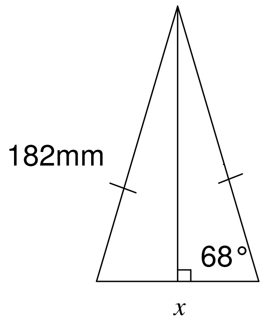
C: $x < -\frac{1}{2}$ ✓

$-(x-1) = -(2x+1)-7$
$-x+1 = -2x-1-7$
$\underline{x = -9}$ ✓✓

Question 2. **[2+3=5 marks]**

Determine the value of x in each of the following:

a)

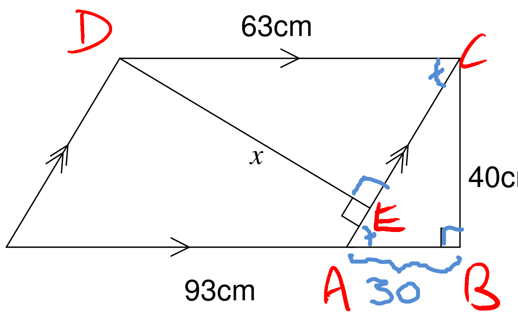


$\cos 68^\circ = \dfrac{x/2}{182} = \dfrac{x}{364}$

$x = 364 \cos 68^\circ$

$= 136.36\text{ mm}$ ✓✓

b)



$\triangle DCE \sim \triangle CAB$ (AAA)

$AC = \sqrt{40^2 + (93-63)^2}$

$= 50\text{ cm}$ ✓

$\dfrac{x}{40} = \dfrac{63}{50}$

$x = \dfrac{40 \times 63}{50}$

$= 50.4\text{ cm}$ ✓✓

OR find $\angle ACB$ (36.87°) using $\tan^{-1}$

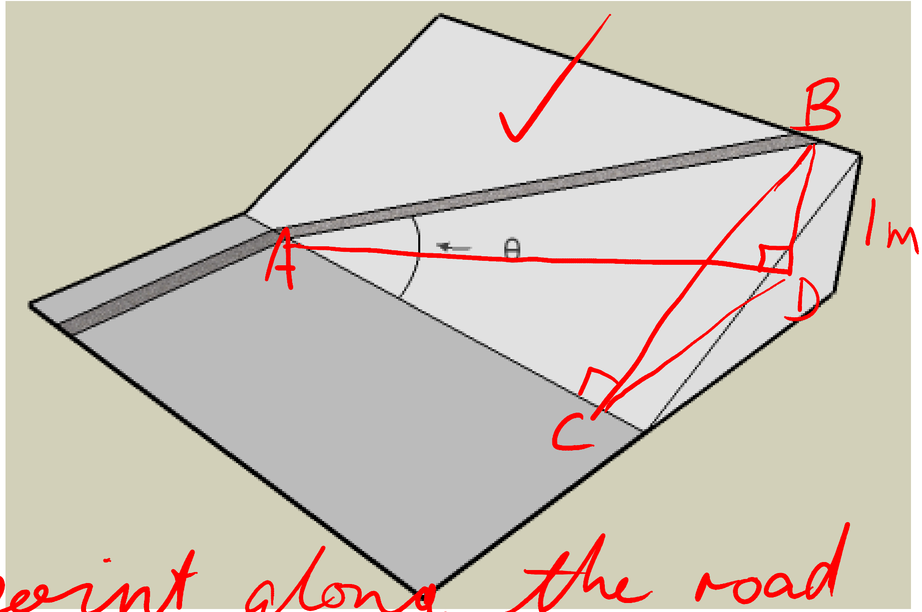
$\angle CDE \cong \angle ACB$

find x using cos.

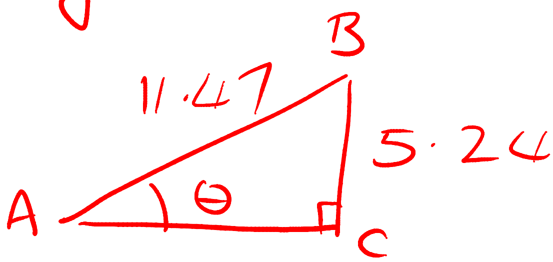
OR construct on classpad.

Question 3. **[5 marks]**

Thompson's Hill is a steep incline. A path of steepest slope going up the hill has an angle of 11° to the horizontal. The Radomiljac Engineering company is tasked with designing a road up the hill that has an angle no greater than 5°. They will achieve this by making the road run up the hill at an angle, as shown. Determine the value of angle θ between the road and the line where the plane of the hill meets the plane of the level ground.



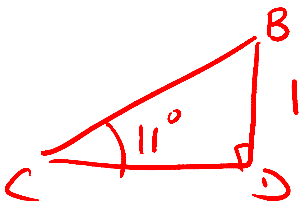
Let B be a point along the road such that BD = 1m.



$$\sin 11° = \frac{1}{BC}$$

$$BC = \frac{1}{\sin 11°}$$

$$= 5.24\text{ m} \checkmark$$



$$AB = \frac{1}{\sin 5°}$$

$$= 11.47\text{m} \checkmark$$

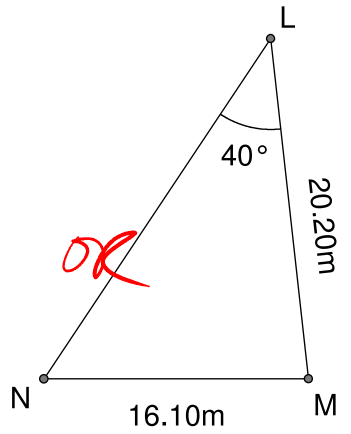
$$\sin\theta = \frac{BC}{AB}$$

$$\theta = \sin^{-1}\frac{BC}{AB}$$

$$= \underline{\underline{27.18°}} \checkmark\checkmark$$

Question 4. **[4 marks]**

Determine the possible values for the area of triangle LMN



$$\frac{\sin \angle N}{20.20} = \frac{\sin 40^\circ}{16.10}$$

$$\sin \angle N = \frac{20.20 \sin 40^\circ}{16.10}$$

$$\angle N = \sin^{-1} 0.806$$

$\angle N = 53.75^\circ$ or 126.25° ✓✓

$\angle M = 86.25^\circ$ or 13.75°

Area $= \frac{1}{2} ab \sin C$

$= \frac{1}{2} \times 20.20 \times 16.10 \sin \angle M$

$= 162.26$ or 38.66 ✓✓

OR

N obtuse: $A = 38.66\ m^2$ ✓✓

N acute: $A = 162.26\ m^2$ ✓✓

((Classpad constr.))

Question 5. **[6 marks]**

The Professor is checking the speed of ocean currents near Gilligan's Island. He knows that the mountain on the island is 1200m high and he measures an angle of elevation and a bearing to the mountain peak while he is sitting on the raft he has had built for the test. Thirty minutes later he measures them again.

	Initial measurements	After thirty minutes
Elevation	21°	27°
Bearing	005°	352°

How fast is the raft drifting?

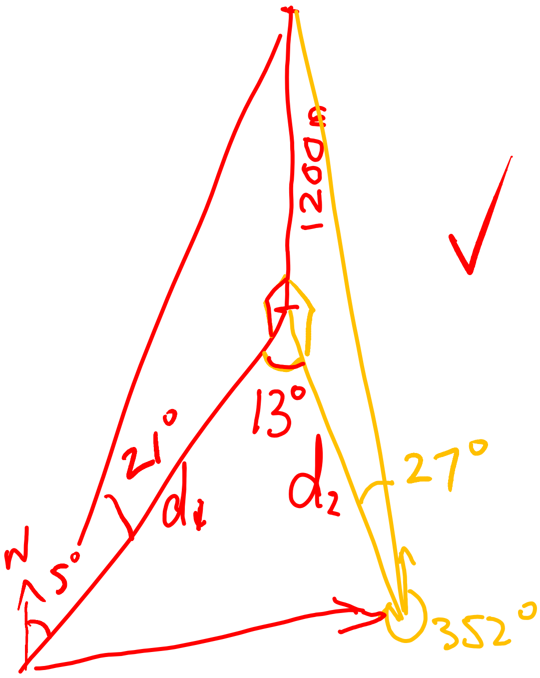
$\tan 21° = \frac{1200}{d_1}$

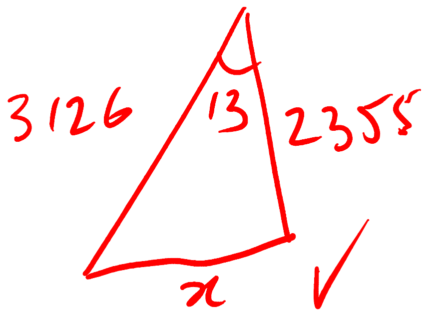
$d_1 = \frac{1200}{\tan 21°}$

$= 3126\text{ m}$ ✓

$d_2 = \frac{1200}{\tan 27°}$

$= 2355\text{ m}$ ✓





$x = \sqrt{3126^2 + 2355^2 - 2 \times 3126 \times 2355 \cos 13}$

$= 985.80$ in 30 min. ✓

$= 1.97\text{ km/h}$ ✓

or 0.55 ms^{-1}

Question 6. **[3 marks]**

Determine the distance along the 32nd south parallel between Perth (32°S, 116°E) and an oil well in the Indian Ocean at (32°S, 57°E). Use R=6380km for the radius of the earth.

$116 - 57 = 59°$ ✓

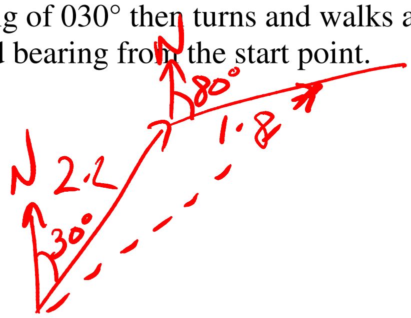
$$d = \frac{59}{360} \times 2\pi R\cos 32°$$

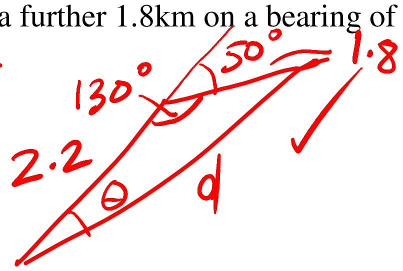
$$= \frac{59}{360} \times 2\pi \times 6380 \cos 32°$$ ✓

$$= 5571.48 \text{ km}$$ ✓

Question 7. **[6 marks]**

A hiker travels 2.2km on a bearing of 030° then turns and walks a further 1.8km on a bearing of 080°. Calculate the hiker's distance and bearing from the start point.





✓

$$d = \sqrt{2.2^2 + 1.8^2 - 2 \times 2.2 \times 1.8 \times \cos 130°}$$

$$= 3.63 \text{ km}$$ ✓✓

$$\frac{\sin\theta}{1.8} = \frac{\sin 130}{d}$$

$$\theta = \sin^{-1} \frac{1.8 \sin 130}{3.63}$$

$$= 22.33°$$ ✓✓

$\therefore$ bearing $= 30 + 22.33° = 52.33°$ ✓

distance $= 3.63$km

Question 8. **[4+4=8 marks]**

In still air an aircraft can fly straight and level at 400km/h. A pilot is planning a course between points A and B where point B is 280km due east of point A. There is a 50km/h wind blowing from the northeast.

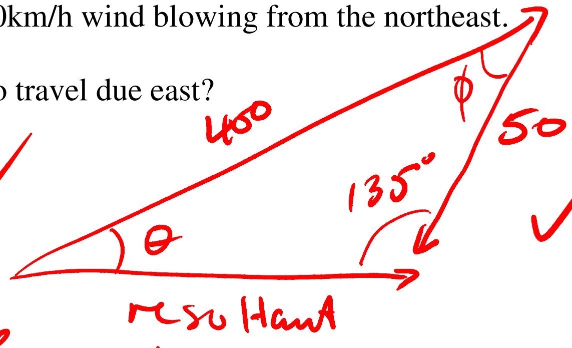
a. In what direction should the aircraft point in order to travel due east?

$$\frac{\sin\theta}{50} = \frac{\sin 135°}{400}$$

$$\theta = \sin^{-1}\frac{50\sin 135°}{400}$$

$$= 5.07°$$ N. of east

bearing = 085°



b. Excluding any time needed for take-off or landing, how long will the trip take? (Give your answer to the nearest minute.)

$$\phi = 180 - 135 - 5.07$$

$$= 39.93°$$

$$\frac{r}{\sin 39.93} = \frac{400}{\sin 135}$$

$$r = 363.08 \text{ km/h}$$

$$\frac{280}{363.08} = 0.77 \text{ h}$$

$$= 46 \text{ min.}$$

Question 9. **[1+3=4 marks]**

Find the values of scalars λ and μ in each of the following cases, given that **a** and **b** are non-parallel vectors.

a. 5λ**a** = 2μ**b**

$\lambda = 0, \quad \mu = 0$

b. λ**a** +3μ**b** = (2+ μ)**a**–(λ–1)**b**

$$\lambda = 2 + \mu$$

$$3\mu = -(\lambda - 1)$$

$$= -(2 + \mu - 1)$$

$$= -\mu - 1$$

$$4\mu = -1$$

$$\mu = -\tfrac{1}{4}$$

$$\lambda = 2 + \mu$$

$$= 1\tfrac{3}{4}$$